Hints to Homework 12

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1 Section 4.9: Antiderivatives

4.9.13 First simplify f as usual to get $f(u) = u^2 + 3u^{-\frac{3}{2}}$ and then antidifferentiate using the fact that the antiderivative of x^r is $\frac{x^{r+1}}{r+1}$ for any real number $r \neq -1$.

4.9.29 Antidifferentiate to get $f(x) = x - 3x^2 + C$, and use $f(1) = 6$ to solve for C.

4.9.47 We are given that $f'(x) = 2x + 1$, so antidifferentiate f, solve for C using $f(1) = 6$, and then find $f(2)$

4.9.52 Think of you being in a car and the graph being your velocity function. Remember that increasing velocity means a concave up position, and decreasing velocity means a concave down position

4.9.64 We are given that $s''(t) = a$, so antidifferentiate twice to get $s(t) = \frac{1}{2}at^2 + Bt + C$, and use $s(0) = s_0$ to solve for C, and $s'(0) = v_0$ to solve for B (you would need to differentiate s to get $s'(t) = at + B$).

4.9.70 The mass is just $F(1)$, where F is the antiderivative of ρ. Solve for C by using the fact that $F(0) = 0$, because a rod of length 0 has mass 0. This will become easier once we do integration, then the mass is just $\int_0^1 \rho(x)dx$.

2 Section 5.1: Areas and distances

5.1.3(a) Just use the definition of right-hand-sums. Sketch the graph and the ractangles, and you'll see whether your estimate is an under or overestimate (if the areas of the rectangles are bigger than the area of the function, then you have an overestimate)

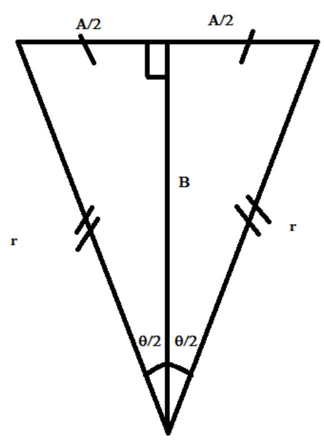
5.1.14 Here you just need to use the fact that $h(t) \approx \sum_{i=0}^{7} \Delta_{t_i} v(t_i)$, where h is the height of the rocket. Use the table to find Δ_{t_i} and $v(t_i)$!

5.1.18 Use the definition of the right-hand-sum. Here $f(x) = \frac{\ln(x)}{x}$, $\frac{b-a}{n} = \frac{7}{n}$ and $x_i = 3 + \frac{7i}{n}$

5.1.20 This is the last problem on the handout about the Area Problem (see bspace or my website). The answer is: It's the area under the graph of $f(x) = x^{10}$ from 5 to 7.

5.1.26 (a) The polygon is made out of n triangles, and each triangle has area (see picture): $\mathcal{A} = \frac{AB}{2} = \frac{r\sin\left(\frac{\theta}{2}\right)\cdot r\cos\left(\frac{\theta}{2}\right)}{2} = \frac{r^2 2\sin\left(2\frac{\theta}{2}\right)}{2} = r^2\sin(\theta) = r^2\sin\left(\frac{2\pi}{n}\right)$), where $\theta = \frac{2\pi}{n}$ (essentially because we are dividing up the whole circle into n pieces). And so, we get: $\boxed{A_n = n\cdot\mathcal{A} = \frac{1}{2}nr^2\sin\left(\frac{2\pi}{n}\right)}$.

1A/Polygon.png



(b). First, use the substitution $t = \frac{2\pi}{n}$ to get $A_n = \frac{2\pi}{t}\frac{1}{2}r^2\sin(t)$ and then let $t \to 0$ and use l'Hopital's rule, remembering that r is a **CONSTANT**!!!!!!!!!

3 Section 5.2: The definite integral

5.2.5(a) This is just the right endpoint method.

5.2.9 This is just the midpoint rule.

5.2.22 Use theorem 4 and the fact that $\Delta x_i = \frac{3}{n}$ and $x_i = 1 + \frac{3i}{n}$, as well as the fact that $\sum_{i=1}^{n} i = \frac{(n)(n+1)}{2}$ and $\sum_{i=1}^{n} i^2 = \frac{(n)(n+1)(2n+1)}{6}$

5.2.30 Use Theorem 4, with $\Delta x_i = \frac{9}{n}$ and $x_i = 1+\frac{9i}{n}$ and $f(x) = x - 4\ln(x)$.

5.2.34 The integral equals: Area of big triangle - Area of semi-circle + Area of small triangle = $4 - 2\pi + 0.5 = 4.5 - 2\pi$

5.2.48 Use the fact that $\int_1^5 f(x)dx = \int_1^4 f(x)dx + \int_4^5 f(x)dx$

5.2.51 Use the fact that $m \leq f(x) \leq M$, and integrate from 0 to 2 (use Property 8 on page 375 of the book)

5.2.59 Use the fact that $e^{-2} \leq e^{-x} \leq 1$ (because e^{-x} is decreasing on $[0,2]$), and so $\frac{x}{e^2} \leq xe^{-x} \leq x$, and now integrate from 0 to 2 (use the property that if $f(x) \leq g(x) \leq h(x)$, then $\int_0^2 f(x)dx \leq \int_0^2 g(x)dx \leq \int_0^2 h(x)dx$

5.2.65 Use the hint and integrate from a to b. You should get: -$\int_a^b |f(x)|dx \leq \int_a^b f(x)dx \leq \int_a^b |f(x)|dx$, but this precisely says that $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$ (in general, if M is nonnegative, then $-M \leq x \leq M$ is the same as saying $|x| \leq M$.

5.2.68 This problem is the hardest one of the whole problem set. Don't worry too much about it (I won't be mad if you don't do it). The hint basically solves it for you. For every N, let $x_1^* = \frac{1}{N^2} \in [x_0, x_1]$. Then $\Delta x = \frac{1}{N}$ and $f(x_1^*) = N^2$, and $\Delta x \cdot f(x_1^*) = N^2$, which goes to ∞ as N goes to ∞. The point is that, to show that something is **NOT** Riemann integrable, you basically need to choose a partition that 'fails' (or two partitions that give you a different answer, see 5.2.67)

4 Section 5.3: The Fundamental Theorem of Calculus

5.3.9 This is just a quick application of Part I of the FTC, except that here y is your variable, **NOT** x. This is just to scare you! The answer is $y^2\sin(y)$.

5.3.16 Hopefully I'll teach you on Tuesday how to do this. The answer is $-\sin(x)(1+\cos^2(x))^{10}$

5.3.23 Straightforward application of the FTC Part II.

5.3.32 Ditto! The antiderivative of $sec^2(t)$ is $\tan(t) + C$.

5.3.44 The function is **NOT** continuous at 0. The FTC Part II only applies for **CONTINUOUS** functions!

5.3.51 The integral is the difference of the area under the graph of x^3 from 0 to 2 and the area under the same graph from -1 to 0